Progression & Series

Arithematic Prog. —

First Term = a Common diff. = d.

Prog : a, a+d, a+2d, ...

General Term : $\boxed{T_n = a + (n-1)d}$

Sum of 'n' term : $\boxed{S_n = \left(\frac{n}{2}\right)\left(2a+(n-1)d\right)}$

$$= \left(\frac{n}{2}\right)(\underbrace{a}_{\text{first term}} + \underbrace{l}_{\text{last term}})$$

$\boxed{T_n = S_n - S_{n-1}}$

Geometric Prog. —

First Term = a Common Ratio = r

Prog : a, ar, ar^2, ...

General Term : $\boxed{T_n = ar^{n-1}}$

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Sum of 'n' terms : $\boxed{S_n = a\left(\frac{r^n - 1}{r - 1}\right)}$

If $|r| < 1 \Rightarrow$ $\boxed{S_\infty = \left(\frac{a}{1-r}\right)}$

☆ $a_1, a_2, a_3, \ldots$ is G.P $\Leftrightarrow$ $\ln(a_1), \ln(a_2), \ldots$ is A.P.

Harmonic Prog.

$a_1, a_2, a_3 \ldots$ in H.P. $\Leftrightarrow$ $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ in A.P.

AM, GM, HM ~~Inequality~~ —

For $x_1, x_2, \ldots, x_n$;

$$AM = \left(\frac{1}{n}\right) \sum x_i \, , \quad GM = \left(\prod x_i\right)^{1/n} , \quad HM = \frac{n}{\sum (1/x_i)}$$

Weighted Means —

$$AM^* = \left(\frac{1}{\sum f_i}\right)\left(\sum f_i x_i\right) , \quad GM^* = \left(\prod x_i^{f_i}\right)^{\frac{1}{\sum f_i}}$$

$$HM^* = \frac{\left(\sum f_i\right)}{\sum (f_i/x_i)}$$

Insertion of Means –

1) $a, A_1, A_2, \ldots, A_n, b$ in A.P.

$\Rightarrow \boxed{d = \left(\dfrac{b-a}{n+1}\right)} \Rightarrow \boxed{\sum A_i = \left(\dfrac{n}{2}\right)(a+b)}$

$\left(\sum A_i = n(AM)\right)$

2) $a, G_1, G_2, \ldots, G_n, b$ in G.P.

$\Rightarrow \boxed{r = \left(\dfrac{b}{a}\right)^{\frac{1}{n+1}}} \Rightarrow \boxed{\prod G_i = (ab)^{(n/2)}}$

$\left(\prod G_i = (GM)^n\right)$

3) $a, H_1, H_2, \ldots, H_n, b$ in H.P.

(Use formulae for A.P.) $\left(\sum \dfrac{1}{H_i} = \dfrac{n}{HM}\right)$

Q) Sum of 2 nos = 13/6. Even no. of AMs inserted b/w them s.t. their sum exceeds their no. by 1. Find no. of means inserted.

A) $(A_1 + \ldots + A_{2n}) = \left(\dfrac{2n}{2}\right)\left(\dfrac{13}{6}\right) = 2n+1 \Rightarrow 2n = 12$

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Q) $a, b \in \mathbb{R}^+$.

AP: a, A_1, A_2, b
GP: a, G_1, G_2, b
HP: a, H_1, H_2, b.

P.t. $\dfrac{G_1 G_2}{H_1 H_2} = \dfrac{A_1 + A_2}{H_1 + H_2} = \dfrac{(2a+b)(a+2b)}{9ab}$

A) $A_1 = \left(\dfrac{2a+b}{3}\right), \quad A_2 = \left(\dfrac{a+2b}{3}\right)$

$G_1 = (a^2b)^{1/3}, \quad G_2 = (ab^2)^{1/3}$

$\dfrac{1}{H_1} = \dfrac{2(1/a)+(1/b)}{3}, \quad \dfrac{1}{H_2} = \dfrac{(1/a)+(2/b)}{3}$

$\Rightarrow H_1 = \left(\dfrac{3ab}{2b+a}\right), \quad H_2 = \left(\dfrac{3ab}{2a+b}\right)$

$\dfrac{G_1 G_2}{H_1 H_2} = \dfrac{(a^2b \cdot ab^2)^{1/3}}{\left(\dfrac{3ab}{2b+a}\right)\left(\dfrac{3ab}{2a+b}\right)} = \dfrac{(2a+b)(a+2b)}{9ab}$

$\dfrac{A_1 + A_2}{H_1 + H_2} = \dfrac{a+b}{\left(\dfrac{3ab}{2b+a}\right)+\left(\dfrac{3ab}{2a+b}\right)} = \dfrac{(2a+b)(a+2b)}{9ab}$

Q) If $S_1, S_2, \ldots S_n$ are sums of ∞ G.Ps. whose first terms are $1, 2, \ldots, n$ and common ratios are $1/2, 1/3, \ldots, 1/(n+1)$ resp.; then find

$$S_1^2 + S_2^2 + \ldots + S_{2n-1}^2$$

A) $S_K = \dfrac{K}{1-\frac{1}{(k+1)}} = \dfrac{k(k+1)}{(k+1)-1} \Rightarrow S_K = (k+1)$

$$\Rightarrow \sum_{k=1}^{2n-1} (S_k^2) = \sum_{k=1}^{2n-1} ((k+1)^2) = \sum_{k=1}^{2n} (k^2) - 1$$

$$= \frac{(2n)(2n+1)(4n+1)}{6} - 1 = \boxed{\frac{n(2n+1)(4n+1)-3}{3}}$$

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AM, GM, HM Inequality

For (+ve) $\mathbb{R}$ nos.,

$$\boxed{AM \geq GM \geq HM}$$

Equality holds when all nos. equal.

Also, $AM^* \geq GM^* \geq HM^*$.

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Q) $x, y, z \in \mathbb{R}^+$, $x+y+z=a$. P.t. $\sum\left(\frac{1}{x}\right) \geq \frac{9}{a}$

A) By AM $\geq$ HM, $\frac{\sum x}{3} \geq \frac{3}{\sum\left(\frac{1}{x}\right)}$

$$\Rightarrow \boxed{\sum\left(\frac{1}{x}\right) \geq \frac{9}{a}}$$

Q) $a, b, c \in \mathbb{R}^+$, $\sum a = 18$. Find max. $(a^2 b^3 c^4)$

A) By AM* $\geq$ GM*, $\boxed{\frac{2(a/2)+3(b/3)+4(c/4)}{2+3+4}} \geq \left[\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{4}\right)^4\right]^{1/9}$

$$\Rightarrow \left(\frac{a+b+c}{9}\right) \geq \left(\frac{a^2b^3c^4}{2^{10}\cdot 3^3}\right)^{1/9}$$

$$\Rightarrow \boxed{a^2b^3c^4 \leq 2^{19}\cdot 3^3}$$

Power Mean Inequality

Let $WPM(m) = \left(\frac{\sum f_i x_i^m}{\sum f_i}\right)^{1/m}$. Then

$$\boxed{WPM(p) > WPM(q)} \quad \text{if } p>q$$

for all $p, q \in \mathbb{R}$

Questions

1) i) $\sum a = 5$, $\max(ab^3c) = ?$

ii) $x^2y^3 = 6$, $\min(3x+4y) = ?$

iii) $\min\left\{\left(\sum a\right)\left(\sum\left(\frac{1}{a}\right)\right)\right\} = ?$

A) i) $\frac{a + 3(b/3) + c}{5} \geq \left(\frac{ab^3c}{3^3}\right)^{1/5} \Rightarrow \boxed{ab^3c \leq 27}$

ii) $\cancel{\frac{3x+4y}{7} \geq (x^3y^4)^{1/7} \Rightarrow 3x+4y \geq}$

$\frac{2(3x/2) + 3(4y/3)}{5} \geq \left[\frac{3^2x^2 \cdot 4^3y^3}{2^2 \cdot 3^3}\right]^{1/5} = \left[\frac{2^6 \cdot 2 \cdot 3}{2^2 \cdot 3}\right]^{1/5}$

$\Rightarrow \boxed{3x + 4y \geq 10}$

iii) $\frac{\sum a}{3} \geq \frac{3}{\sum(1/a)} \Rightarrow \boxed{\left[\sum(a)\right]\left[\sum\left(\frac{1}{a}\right)\right] \geq 9}$

2) $a_1 + \cdots + a_{50} = 50$, $\min\left\{\frac{1}{a_1} + \cdots + \frac{1}{a_{50}}\right\} = ?$

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A) $\left(\sum a_i\right)\left(\sum\left(\frac{1}{a_i}\right)\right) \geq 2500 \Rightarrow \boxed{\sum\left(\frac{1}{a_i}\right) \geq 50}$

3) i) $(1+a_1+a_1^2)\ldots(1+a_n+a_n^2) \geq 3^n a_1\ldots a_n$

ii) $\sum x = 3$. P.t. $\sum(1/x) \geq 3$

iii) P.t. $\sum\left(\frac{ab}{a+b}\right) < \frac{\sum a}{2}$

iv) $\sum a^2(1+b^2) > 6abc$

v) $\prod (a+1)^7 > 7^7 a^4b^4c^4$

vi) $3^n \geq 1 + 2n\sqrt{3^{n-1}}$

vii) $\sum a = 18$. P.t. $a^2b^3c^4 \leq 2^{19}\cdot 3^3$

viii) $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ has 4 (+ve) R roots, find a & b.

A) i) $\left(a_i + \frac{1}{a_i} + 1\right) \geq 3 \Rightarrow \boxed{\prod\left(a_i + \frac{1}{a_i} + 1\right) \geq 3^n}$

ii) $\left(\sum x\right)\left(\sum(1/x)\right) \geq 9 \Rightarrow \boxed{\sum(1/x) \geq 3}$

iii) $\frac{2}{1/a+1/b} \leq \frac{a+b}{2} \Rightarrow \boxed{\sum\left(\frac{ab}{a+b}\right) \leq \frac{\sum a}{2}}$

iv) $a^2+b^2+c^2+(abc)^2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right) \geq 6\left(\frac{a^2b^2c^2\,a^6b^6c^6}{a^2\cdot b^2\cdot c^2}\right)^{1/6}$

$\Rightarrow \boxed{\sum\left(a^2(1+b^2)\right) \geq 6abc}$

v) $\prod(a+1) = (abc+ab+bc+ca+a+b+c)+1 \geq 1+7(abc)^{4/7}$

$\Rightarrow \boxed{\prod(a+1)^7 \geq 7^7(abc)^{4/7}}$

vi) $\frac{1+3+\dots+3^{n-1}}{n} \geq \left(3^{\frac{n(n-1)}{2}}\right)^{1/n} \Rightarrow \boxed{3^n \geq 1+2n\sqrt{3^{n-1}}}$

vii) $\frac{2(a/2)+3(b/3)+4(c/4)}{9} \geq \left(\frac{a^2\cdot b^3\cdot c^4}{2^2\cdot 3^3\cdot 2^8}\right)^{1/9}$

$\Rightarrow \boxed{a^2b^3c^4 \leq 2^{19}\cdot 3^3}$

viii) $\frac{\sum\alpha}{4} = 1, \quad \left(\prod\alpha\right)^{1/4} = 1 \Rightarrow AM = GM \Rightarrow \alpha=\beta=\gamma=\delta$

$\Rightarrow \boxed{a=6, \quad b=(-4)}$

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4) $\prod a_i = c \qquad \min(a_1 + \dots + a_{n-1} + 2a_n) = ?$

A) $$\frac{a_1 + \dots + a_{n-1} + 2a_n}{n} \geq (2a_1 \dots a_n)^{1/n}$$

$$\Rightarrow \boxed{(a_1 + \dots + a_{n-1} + 2a_n) \geq n(2c)^{1/n}}$$

5) If $\alpha \in (0, \pi/2)$. find min. of $\sqrt{x^2+x} + \frac{t_\alpha^2}{\sqrt{x^2+x}}$

A) $$\text{Req.} \geq 2(t_\alpha^2)^{1/2} = 2|t_\alpha|$$

6) $\sum a_n = 1$ P.t. $\sum \left(a_n + \frac{1}{a_n}\right)^2 \geq \frac{(1+n^2)^2}{n}$

A) $$\left(\frac{\sum (a_n + 1/a_n)^2}{n}\right)^{1/2} \geq \left(\frac{\sum a_n + \sum 1/a_n}{n}\right)$$

Also, $$\frac{\sum a_n}{n} \geq \frac{n}{\sum 1/a_n} \Rightarrow \sum 1/a_n \geq n^2$$

$$\Rightarrow \left(\frac{\sum (a_n + 1/a_n)}{n}\right)^{1/2} \geq \left(\frac{\sum a_n + n^2}{n}\right) = \left(\frac{1+n^2}{n}\right)$$

$$\Rightarrow \boxed{\sum \left(a_n + \frac{1}{a_n}\right)^2 \geq \frac{(1+n^2)}{n}}$$

7) a, b, c in G.P P.t. $a^2 + 2bc - 3ac > 0$

A) Let $a, b, c \equiv a, ar, ar^2 \Rightarrow a^2 + 2a^2r^3 - 3ar^2 > 0$

Now, $\left(\frac{r+r+1/r^2}{3}\right) \geq 1 \Rightarrow 2r^3 + 1 \geq 3r^2$

$\Rightarrow \left(2r + \frac{1}{r^2}\right) \geq 3$

8) $\sum x^2 = 1$. $\max(\cancel{x^2+y^3}, x^2y^3z^4) = ?$

A) $\left(\frac{x^2 + (3/2)(2y^2/3) + 2(z^2/2)}{1 + 3/2 + 2}\right) \geq \left(x^2 \cdot \left(\frac{2}{3}\right)^{3/2} \cdot y^3 \cdot \left(\frac{1}{2}\right)^2 \cdot z^4\right)^{\frac{2}{9}}$

$\Rightarrow \left(\frac{2}{9}\right) \geq \boxed{x^2y^3z^4 \cdot \frac{1}{3^{3/2}} \cdot \frac{1}{2^{1/2}}}^{2/9}$

$\Rightarrow x^2y^3z^4 \leq \cancel{2^{3/9}} \frac{2^{9/2} \cdot 3^{3/2} \cdot 2^{1/2}}{3^9} \Rightarrow x^2y^3z^4 \leq$

9) x, y, z in H.P. P.t. $ze^{x-y} + xe^{z-y} \geq \left(\frac{2xz}{y}\right)$

A) $\left(\frac{ze^{x-y} + xe^{z-y}}{x+z}\right) \geq \left(e^{2x-yz+xz-xy}\right)^{\frac{1}{x+z}} = \left(e^{(xyz)\left(\frac{2}{y} - \frac{1}{x} - \frac{1}{z}\right)}\right)^{\frac{1}{x+z}}$

$\Rightarrow ze^{x-y} + xe^{z-y} \geq (x+z) \Rightarrow \boxed{ze^{x-y} + xe^{z-y} \geq \left(\frac{2xz}{y}\right)}$

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10) A_i, H_i, G_i are 'n' AMs, HMs & GMs b/w 2 nos. P.t.

$$\left(\frac{\sum A_n}{n}\right) \geq \left[\frac{(\prod G_n)^2}{\prod H_n}\right]^{1/n}$$

A) Observe, $\left(\frac{\sum A_n}{n}\right) = AM$, $\left(\prod G_i\right)^{\frac{2}{n}} = G.M$

and $\frac{n}{HM} = \sum\left(\frac{1}{H_i}\right)$.

Now, $\left(\prod H_i\right)^{1/n} \geq \frac{n}{\sum\left(\frac{1}{H_i}\right)} = HM$

$\Rightarrow \frac{1}{H.M} \geq \frac{1}{(\prod H_i)^{1/n}} \Rightarrow \left(\frac{GM}{HM}\right) \geq \left(\frac{(\prod G_i)^2}{(\prod H_i)}\right)^{1/n}$

$\Rightarrow (AM) \geq \left(\frac{(\prod G_i)^2}{(\prod H_i)}\right)^{1/n} \Rightarrow \boxed{\left(\frac{\sum A_i}{n}\right) \geq \left(\frac{(\prod G_i)^2}{(\prod H_i)}\right)^{1/n}}$

11) P.t. $\left(\frac{a^2+b^2+c^2}{a+b+c}\right)^{(a+b+c)} \geq a^a b^b c^c$

A) $\left(\frac{a\cdot a + b\cdot b + c\cdot c}{a+b+c}\right)^{(a+b+c)} \geq \left(a^a b^b c^c\right)$

12) a, b, c in HP. P.t. $\left(\frac{a+b}{2a-b}\right)+\left(\frac{c+b}{2c-b}\right) > 4$

A) $\left(\frac{1/a+1/b}{2/b-1/a}\right)+\left(\frac{1/b+1/c}{2/b-1/c}\right)=\left(\frac{c}{a}+\frac{c}{b}+\frac{a}{b}+\frac{a}{c}\right)$

$$= \left(\frac{c}{a}+\frac{a}{c}\right)+\left(\frac{a+c}{2}\right)\left(\frac{1}{a}+\frac{1}{c}\right) > 2+2 = 4$$

13) (Same as Q3)(iv))

14) P.t. $(a^n - b^n) \geq n(ab)^{\left(\frac{n-1}{2}\right)}(a-b)$

A) $\left(\frac{a^n-b^n}{a-b}\right) = \left(a^{n-1}+a^{n-2}b+\ldots+ab^{n-2}+b^{n-1}\right) \geq n\left(a^{\frac{n(n-1)}{2}}\, b^{\frac{n(n-1)}{2}}\right)^{1/n}$

$$\Rightarrow \boxed{(a^n-b^n) \geq n(ab)^{\left(\frac{n-1}{2}\right)}(a-b)}$$

15) P.t. $1^m + 3^m + \ldots + (2n-1)^m > n^{m+1}$

A) $\left(\frac{1^m+\ldots+(2n-1)^m}{n}\right)^{1/m} > \left(\frac{1+\ldots+(2n-1)}{n}\right) = n$

$$\Rightarrow \boxed{1^m+\ldots+(2n-1)^m > n^{m+1}}$$

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Imp. Series —

1) $\sum_{k=1}^{n} (k) = \frac{n(n+1)}{2}$

2) $\sum_{k=1}^{n} (k^2) = \frac{n(n+1)(2n+1)}{6}$

3) $\sum_{k=1}^{n} (k^3) = \left(\frac{n(n+1)}{2}\right)^2$

4) $\sum_{k=1}^{n} (1) = n$

Arithemetico Geometric Prog. —

$ab, \; (a+d)br, \; (a+2d)br^2, \; \ldots$

Eg:

$$S = 1\cdot 2 + 2\cdot 2^2 + 3\cdot 2^3 + \ldots + 100\cdot 2^{100}$$

$$2S = 1\cdot 2^2 + 2\cdot 2^3 + \ldots + 99\cdot 2^{100} + 100\cdot 2^{101}$$

$$\Rightarrow (-S) = 1\cdot 2 + 1\cdot 2^2 + 1\cdot 2^3 + \ldots + 1\cdot 2^{100} - 100\cdot 2^{101}$$

$$\Rightarrow (-S) = (2)\left(\frac{2^{100}-1}{2-1}\right) - 100\cdot 2^{101}$$

$$\Rightarrow \boxed{S = 198\cdot 2^{100} + 2}$$

Method of Difference –

Q) Find the sum $5+7+11+17+25+\ldots$

A) $\cancel{S = 5 + 7 + 1}$

$$S = 5+7+11+\ldots+t_n$$
$$S = \quad 5+7+\ldots+t_{n-1}+t_n$$

$$\Rightarrow \quad t_n = 5+\left(2+4+6+\ldots+(t_n-t_{n-1})\right)$$

$$\Rightarrow \quad t_n = 5+\left(\frac{n-1}{2}\right)\left(2\cdot 2+(n-1-1)\cdot 2\right)$$

$$\Rightarrow \quad t_n = 5+2(n-1)+(n-1)(n-2)$$

Now, find S by $\sum t_n$.

Telescopic Series –

Q) $\displaystyle\sum_{k=1}^{n}\left(k(k+1)(k+2)\right)$.

$$\frac{k(k+1)(k+2)\left[(k+3)-(k-1)\right]}{4}$$

A) $$\sum_{k=1}^{n}\left(k(k+1)(k+2)\right) = \left(\frac{-1}{4}\right)\sum_{k=1}^{n}\left[(k-1)k(k+1) - k(k+1)(k+2)\right]$$

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$$= \left(\frac{1}{4}\right)\begin{bmatrix} 1\cdot 2\cdot 3 - 0\cdot 1\cdot 2 \\ + 2\cdot 3\cdot 4 - 1\cdot 2\cdot 3 \\ \vdots \\ + (n+2)(n+1)n - (n+1)n(n-1) \end{bmatrix}$$

$$= \boxed{\left(\frac{n(n+1)(n+2)}{4}\right)}$$

☆Q) $\sum_{k=1}^{n}\left(\frac{1}{k(k+1)(k+2)}\right)$.

$$\left(\frac{1}{2}\right)\left[\frac{(k+2)-(k)}{k(k+1)(k+2)}\right]$$

A) $$\sum_{k=1}^{n}\left(\frac{1}{k(k+1)(k+2)}\right) = \left(\frac{1}{2}\right)\sum_{k=1}^{n}\left(\frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)}\right)$$

$$= \left(\frac{1}{2}\right)\begin{bmatrix} 1/1\cdot 2 - 1/2\cdot 3 \\ + 1/2\cdot 3 - 1/3\cdot 4 \\ \vdots \\ + 1/n(n+1) - 1/(n+1)(n+2) \end{bmatrix}$$

$$= \boxed{\frac{(n+1)(n+2)-1}{2(n+1)(n+2)}}$$

Q) $$S = \left(\frac{1}{1+1^2+1^4}\right) + \left(\frac{2}{1+2^2+2^4}\right) + \left(\frac{3}{1+3^2+3^4}\right) + \dots \infty$$

A) $$T_n = \left(\frac{n}{1+n^2+n^4}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{n(n-1)+1} - \frac{1}{(n+1)n+1}\right)$$

$$\Rightarrow S = \sum T_n = \left(\frac{1}{2}\right)\left[\frac{1}{1\cdot 0+1} - \frac{1}{2\cdot 1+1} + \frac{1}{2\cdot 1+1} - \frac{1}{3\cdot 2+1} + \dots\right]$$

$$\Rightarrow \boxed{S = 1/2}$$

Q) If $\sum_{k=1}^{n} (T_k) = \frac{n(n+1)(n+2)(n+3)}{8}$, $\sum_{k=1}^{n}\left(\frac{1}{T_k}\right) = ?$

A) $T_n = \sum_{k=1}^{n}(T_k) - \sum_{k=1}^{n-1}(T_k) = \frac{n(n+1)(n+2)}{8}\left[(n+3)-(n-1)\right]$

$$\Rightarrow T_k = \frac{k(k+1)(k+2)}{2}$$

$$\sum_{k=1}^{n}\left(1/T_k\right) = \sum_{k=1}^{n}\left(\frac{2}{k(k+1)(k+2)}\right) = \sum_{k=1}^{n}\left(\frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)}\right)$$

$$= \left(\left(\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3}\right) + \left(\frac{1}{2\cdot 3} - \frac{1}{3\cdot 4}\right) + \dots + \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right)\right)$$

$$= \boxed{\frac{1}{2} - \frac{1}{(n+1)(n+2)}}$$

Q) i) $\frac{1}{1\cdot 3} + \frac{2}{1\cdot 3\cdot 5} + \frac{3}{1\cdot 3\cdot 5\cdot 7} + \frac{4}{1\cdot 3\cdot 5\cdot 7\cdot 9} + \dots$ upto 'n' terms.

ii) $\frac{1}{3} + \frac{3}{3\cdot 7} + \frac{5}{3\cdot 7\cdot 11} + \frac{7}{3\cdot 7\cdot 11\cdot 15} + \dots$ upto 'n' terms.

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A) i) $$\sum_{k=1}^{n}\left(\frac{k}{1\cdot 3\cdot\ldots\cdot(2k+1)}\right)=\left(\frac{1}{2}\right)\sum_{k=1}^{n}\left(\frac{(2k+1)-1}{1\cdot 3\cdot\ldots\cdot(2k+1)}\right)$$

$$=\left(\frac{1}{2}\right)\sum_{k=1}^{n}\left(\frac{1}{1\cdot\ldots\cdot(2k-1)}-\frac{1}{1\cdot\ldots\cdot(2k+1)}\right)=\left(\frac{1}{2}\right)\left|\begin{array}{l} 1/1-\cancel{1/1\cdot 3} \\ +\cancel{1/1\cdot 3}-\cancel{1/3\cdot 5\cdot 1} \\ \quad\vdots \\ +\cancel{\frac{1}{1\cdot\ldots\cdot(2n-1)}}-\frac{1}{1\cdot\ldots\cdot(2n+1)}\end{array}\right.$$

$$=\boxed{\left(\frac{1}{2}\right)\left[1-\frac{1}{1\cdot 3\cdot\ldots\cdot(2n+1)}\right]}$$

ii) $$\sum_{k=1}^{n}\left(\frac{(2k-1)}{3\cdot 7\cdot\ldots\cdot(4k-1)}\right)=\left(\frac{1}{2}\right)\sum_{k=1}^{n}\left(\frac{(4k-1)-1}{3\cdot 7\cdot\ldots\cdot(4k-1)}\right)$$

$(+1/3)$

$$=\left(\frac{1}{2}\right)\sum_{k=2}^{n}\left(\frac{1}{3\cdot\ldots\cdot(4k-5)}-\frac{1}{3\cdot\ldots\cdot(4k-1)}\right)=\left(\frac{1}{2}\right)\left|\begin{array}{l} 1/3-\cancel{1/3\cdot 7} \\ +\cancel{1/3\cdot 7}-\cancel{1/3\cdot 7\cdot 11} \\ \quad\vdots \\ +\cancel{\frac{1}{3\cdot\ldots\cdot(4n-5)}}-\frac{1}{3\cdot\ldots\cdot(4n-1)}\end{array}\right.+(1/3)$$

$$=\boxed{\frac{1}{3}+\frac{1}{2}\left[\frac{1}{3}-\frac{1}{3\cdot 7\cdot\ldots\cdot(4n-1)}\right]}$$